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Sue Kelley
Temple University, United States

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Identifying Student Responsibilities While Watching Mathematics Instructional Videos

Sue Kelley

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Abstract

This study investigated the ways in which college algebra students watch mathematics instructional videos about completing the square with the goal of identifying student responsibilities within a particular video and across different videos. Guided by the theory of didactic situations that has defined implicit teacher and student responsibilities within the context of the face-to-face mathematics classroom, participants watched three different videos about completing the square and answered interview questions. Using categories previously identified by the didactic contract for the face-to-face classroom, this study expanded the types of student responsibilities identified specifically for video watching and found that participants, regardless of overall prior knowledge but who had prior knowledge of completing the square, held a responsibility to use the specific set of steps they were taught by their teacher to solve problems. Findings may be useful to both mathematics teachers and video creators.

Introduction

Digital instructional videos on a multitude of mathematical topics have become increasingly accessible to both teachers and students. Videos have made their way into the mathematics classroom in the form of instruction (Trenholm et al., 2012; Zhang et al., 2006), remediation (Pinzon et al., 2016), and motivation (de Araujo et al., 2017a; Guo et al., 2014; Sahin et al., 2015). Additionally, the way we think about mathematics instruction and the use of classroom time has begun to change through ideas such as the flipped or blended classroom and video is a large part of this change (Bergmann & Sams, 2007; Bowers et al., 2012; Kirvan et al., 2015; Pinzon et al., 2016). Video has been used to move mathematics instruction into a just-in-time or personalized format in the form of fully online classes and Massive Online Open Courses (MOOCs) (Hegeman, 2015; Zhang et al., 2006). Despite its extensive use, the instructional value of video to improve student learning in mathematics is still mixed (Crook & Schofield, 2017; Trenholm et al., 2012). Research suggests that the instructional style of videos has impacts on student learning (Brodahl & Wathne, 2016; de Araujo et al., 2017b; Ilioudi et al., 2013).

While studies have investigated the results of student learning gains and attitudes following video watching, a need remains for more research related to how students are interacting with videos for mathematics learning. In order to better understand what students are doing as they watch videos to learn mathematics, I suggest that the

Theory of Didactic Situations (TDS) and its counterpart, the Documentational Approach to Didactics (DAD) will aide in the framing of the responsibilities students have related to mathematics learning. The didactic contract has been used to consider what the student is responsible for in terms of learning as well as what the teacher is responsible for within the mathematics classroom (Perrin-Glorian & Hersant, 2003). I will use these ideas to understand how the responsibilities held by students in college algebra may impact their learning from mathematics instructional videos. In the process, I will identify responsibilities that students hold for themselves, their teachers and the video. The research questions for this study are:

1. How are student responsibilities similar or different among students within a particular video?
2. How are student responsibilities similar or different among students with different levels of prior knowledge related to solving quadratic functions using completing the square?

Previous Literature Related to Video

Videos can be used for mathematics instruction and learning in a variety of settings. For this study I defined video within the context of mathematics instructional videos that are usually created by an instructor who is solving problems in a screencast recording and are widely available to students in YouTube. I will define and discuss the use of video within three instructional settings: the face-to-face classroom, fully online instruction that includes MOOCs, and the flipped or blended classroom. Each of these instructional settings has a wide variety of teaching and learning ideas and constructs that could be discussed. This paper will be limited to those facets of each setting that relate directly to video watching. Other ideas will only be addressed as they relate to video watching in these contexts.

Within the face-to-face classroom, video can be used in a variety of ways to remediate and fill in learning gaps for students in a traditional class. Higher and more sustained levels of student engagement were identified in courses where embedded videos were accessible to students for just-in-time mathematics remediation in introductory geoscience classes (Burn et al., 2013). Video can also be used to archive class lectures in both hybrid and face-to-face classes. Using both class screen captures with audio and video taping regular class lectures, researchers found that archived videos added value to the learning process with little effort on the part of the instructor (Cascaval, Fogler, Abrams, and Durham (2008). Other studies suggest that increased student engagement and enjoyment occurred with videos that give students the opportunity to review concepts and learn at their own pace (Ahmad et al., 2013; Kay, 2014).

Video is also useful for instruction in fully online courses including MOOCs. Because students are separated from teachers in space and sometimes in time within online courses, the video may be the only form of explicit content delivery within the course (Trenholm et al., 2012). These instructional videos, often called e-lectures, can be formatted in several different ways including as a recorded lecture, a voice-over presentation or as a picture-in-picture presentation (Crook & Schofield, 2017; de Araujo et al., 2017a).

Another instructional setting for video use is the flipped or blended classroom. The flipped class is defined as a class where the activities typically done in class are now done at home, and the tasks generally thought of as homework are now done during the class (Bergmann & Sams, 2007). Within the context of a traditional, face-to-face mathematics classroom, flipping the class means that the teacher creates their own videos or assigns some other instructional video for students to watch at home (Pinzon et al., 2016). While variety of variables and research questions related to the use of video in the flipped classroom have been investigated, I will focus on those related to the nature of videos and their impact on student understanding.

Just having a flipped classroom does not guarantee increased conceptual understanding for students. Videos used in flipped mathematics classes are often procedural in nature and may not provide full understanding of the mathematics involved (Kirvan et al., 2015). Although the classroom activities may add a conceptual perspective to the learning, the nature of the mathematics within the videos is also an important consideration. Bowers, Passentino, and Connors (2012) used design research with instructional design specialists and preservice secondary teachers to identify aspects of a video along with an interactive applet to support the reinterpretation of thinking about mathematics. Because the nature of mathematics includes both procedural and conceptual knowledge (Star, 2005), videos used for teaching mathematics must also include both types of mathematical instruction. Bowers, Passentino, and Connors (2012) suggest that one way to improve mathematics instructional videos is to include more opportunities for students to develop conceptual knowledge of mathematics content.

Another variable included in research related to videos used in flipped classrooms is the position of digital video as curriculum materials. de Araujo, Otten, and Birisci (2017b) investigated how a community college professor used digital video as curriculum materials when flipping her College Algebra class for the first time. Using the curriculum framework identified by Remillard and Heck (2014), the researchers described how the professor chose to make videos and found that the textbook materials were supplanted by the videos as curriculum materials during the process of flipping the class. Unlike the position of videos within the traditional classroom, where the instructor is directing instruction within the class and videos are used to supplement this instruction, the position of video in the flipped classroom is more complex.

Because the videos were created by the course instructors they were considered to be curriculum for the class but were also modified through the personality and pedagogy of the instructor. de Araujo et al. (2017b) suggest that the enacted curriculum framework needs to be modified to include the use of videos in a flipped class context. By creating videos that were similar to her live lectures, the professor expressed that she could better meet the instructional needs of her students by providing more extensive explanations and by addressing student misconceptions.

To summarize, video has applications in multiple instructional settings, including the face-to-face classroom, fully online classes and the flipped/blended class. Videos can be used to fill learning gaps (Ahmad et al., 2013; Kay, 2014) or to motivate students to think about mathematics in real world contexts (Petocz & Wood, 2001). In a fully online setting, videos that contain the instructor's voice or face can add presence and decrease transactional distance between the student and teacher (Trenholm et al., 2012). Content instruction can be presented through videos that are watched prior to the live class in order to flip the class format (Pinzon et al., 2016). In all of these settings, videos can be used to increase student learning and to improve student motivation and attitudes toward learning mathematics.

While studies have investigated the results of student learning gains and attitudes following video watching, few studies have actually observed what students are doing during video watching. Weinberg and Thomas (2018) represents one study that did, defining student sense-making activities while watching mathematics videos. Student learning perspectives were categorized as falling into one or more sense-making frames originally identified in Weinberg, Wiesner, and Fukawa-Connelly (2014). Weinberg and Thomas (2018) suggested that students in their study attempted to monitor their own understandings while watching videos but were often unsuccessful.

Additionally, what students learned from the video was related to the questions they were asked after the video, suggesting that giving quizzes during or after video watching may have impacted student learning. Prior knowledge of students was also impactful to student ability to add to their understanding, suggesting that drawing student attention to content and ideas they already knew may have helped students to build on this knowledge while watching the videos. Although this study began to identify ways in which students were using videos, a need remains for more research related to student responsibilities as they interact with videos for mathematics learning.

Theoretical Framework for Research

In this study I used the theory of didactic situations (Brousseau, Berdard, & Nontna, 2014) and the documentational approach to didactics (Gueudet & Pepin, 2018) to identify the rules and responsibilities that participants hold for themselves in instructional situations, generally, and the video they are watching, in particular. I will discuss uses of the didactic contract in prior research and how I suggest that it can be used to describe responsibilities of students while watching mathematics videos.

According to the Theory of Didactic Situations (TDS), a didactic contract is defined as all of the implicit expectations and beliefs between teachers and students in an instructional situation that are deemed appropriate to the mathematics being taught (Brousseau et al., 2014; Gueudet & Pepin, 2018). This contract results from the unspoken negotiations of relationships between a teacher, students, a milieu and a system of education. The didactic contract can also be thought of as the “rules of the game” in a didactic situation (Duah, Croft & Inglis, 2014). These rules, while often implicit have been used to understand the interaction between a teacher and student related to knowledge and tasks around which they are relating and how the rules may determine the behavior and thinking of the participants for this interaction (Elia et al., 2009).

Herbst (2006) used the didactic contract to understand why teachers and students work together and how they divide the labor. He distinguished between the “problem,” “task,” and “situation” (p. 315) as students engaged in solving mathematics problems. In this way of understanding the didactic contract, the problem represented embodiment of the piece of knowledge and stood alone as a mathematical concept independent of the solver. However, as educators, we must understand how the student’s knowing of the knowledge embodied in the problem springs from the interaction with the problem. This knowing was represented by the task that includes the context of the problem and all of the ways in which the student may act on the problem to solve it, including resources and feedback received by the student which may or may not be enacted. Situations had to do with the responsibilities that defined the enactment of different roles, norms and scripts in the mathematics classroom.

The didactic contract is thought about in two parts, one related to the student and one to the teacher. Researchers have applied the didactic contract to the interactions in a typical mathematics classroom. Brousseau, Berdard, and Nontna (2014) identified both didactical and ethical responsibilities that include the responsibility to organize the mathematics in a standard way and the responsibility to act as the authority of mathematical truth in the classroom.

Other teacher responsibilities identified are listed in Table 1. Additionally, student responsibilities defined in prior research include the responsibility to use the data in the problem along with known arithmetic or algebraic operations and algorithms that are combined appropriately to find the answer and the responsibility to give an answer to every problem (Elia et al., 2009). While these responsibilities have been defined for the face-to-face classroom, I suggest that teacher and student responsibilities may be different for students watching mathematics instructional videos.

Table 1. Teacher Responsibilities

The teacher has the responsibility to…
… identify errors that have been or might be made and pass judgment on them (without passing judgment on their authors).
… provide the students with a moderate amount of individual help (as with the natural learning of a language).
… organize the mathematics in the standard way.
… confirm the mathematics or give proofs.
… support the collective and individual activity of the students.
… attest in the end to the truth of the mathematics that has been done.

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Herbst, Nachlieli, and Chazan (2011) used the idea of the didactic contract along with what they define as "practical rationalities" or dispositions of members of a didactic event to act in a certain way to outline what is reasonable or customary in a particular situation to interpret teacher/student interactions in a high school mathematics classroom. They suggest that teaching high school mathematics involves entering into a contract that applies roles to the teacher and students in terms of the mathematics tasks in the classroom. While implicit, Herbst et al. (2011) used the didactic contract to identify the instructional situations within the classroom in which teachers and students work together in order to make sense of the mathematics. In this interpretation the instructional situation involves norms that are understood within the classroom and that regulate the trade of work between the teacher and the student. These norms involve tacit expectations of both the teacher and the student. An example of this in the math classroom would be solving a geometry problem.

According to the didactic contract of the class, the teacher may give information about the mathematics involved in the topic of the day. The practical rationalities of the classroom may dictate that the teacher has the right to pose questions that scaffold the knowledge of students and students are expected to answer. In this way, the work involved in making sense of the mathematics is traded and negotiated between the teacher and student. While the trading of work and the norms related to this symbolic economy in a face-to-face mathematics classroom is defined, the practical rationalities involved in learning mathematics from video watching may involve different dispositions and expectations. In this study I looked at what the participants viewed as the rules that define their responsibilities in learning mathematics from videos as well as their views about responsibilities of the video as a resource for learning mathematics.

Methods

Participants

For this study I recruited participants from 5 sections of the fall semester of a college algebra course taught by four different professors in a large university in the northeastern United States. Students in this course were from a variety of majors and represent a wide distribution of mathematics skill and knowledge. Participants who completed both interviews were compensated with a $20 VISA gift card following completion of the second interview. Of the 38 who consented to participate, one participant withdrew from the study, 14 participants did not respond to scheduling emails, and 23 participants completed both interviews.

Data Collection

Participants were asked to participate in two one-hour interview sessions. During the first session, I administered a pretest about solving quadratic equations and a Mathematics Related Beliefs Questionnaire (MRBQ) (de Corte, 2015). The full text of the pretest and MRBQ can be seen in Appendix A. After testing, participants were asked some general interview questions according to the interview protocol shown in Table 2. These questions were meant to allow me to get to know the participant and elaborate on their mathematics related beliefs and beliefs about video watching.

Table 2. Protocol For Interview 1

How did you like high school math? What was the last high school math course you took? What's the highest level math class you have taken?
What did you struggle with and what did you find confusing?
What does it mean to be good at math? Why?
Would you rather watch live math lecture or a math video? Why? What makes this other one worse?
When do you watch math videos?
What kinds of situations would lead you to search for a video?
Do you use videos to help with very specific questions, or with more general topics, or both?
What kinds of videos do you look for or prefer?
How long do you like them to be?
Do you like when there's lots of explanation, or when they're just completing the problems?
Are there particular websites you regularly use?
What do you find helpful about watching videos?
Do you typically watch videos straight through?
If not, what do you usually do?
Do you pause videos often? If so, when?
Do you rewind videos often? If so, when?
What do you do when you are confused or don't understand something in math? (ask for specific examples)
What do you do when you make mistakes in math?
Do mistakes help you to learn? How (or why not)?

The second interview was devoted to watching the three selected videos, a Khan Academy video, another procedural video by Math Meeting and a geometric representation by MindYourDecisions. More details about the videos used in this study can be found in Appendix B. I asked participants to watch the Khan Academy video first because this type of video was most familiar to them and viewing from more familiar to less familiar seemed least likely to bias participants' evaluation of the videos.

During the video watching, participants were instructed to stop the video when they saw something that caught their attention/interest or when they were confused. When participants stopped the video, I noted the timestamp on the video and asked them why they stopped the video and to describe what was attention grabbing or confusing to them. If participants did not pause, I asked them if there were any parts of the video that they thought about pausing or if they felt confident that they understood everything in the video. After watching each video, I asked the participants a series of questions according to the interview protocol in Table 3.

Table 3. Protocol for Interview 2

What did you notice about the video?
What did you think was important to take away? Why?
What in the video did you notice, but not find valuable? Why?
What would you say is the main point of completing the square?
What does it mean to complete the square?
How do you find the number to add or subtract from the equation?
How did you choose that number?
How did you know how to rewrite the equation with an x plus something in parentheses?
Why do you have to take the square root of both sides?
Why do you need to use both the positive and the negative value of the square root?
Why are there 2 answers to this problem?
What do the answers to this problem represent?

Following the interview, participants were given a problem similar to those in the videos (8x^2+16x-42=0) and were asked to solve the problem to the best of their ability. Participants were also given the option of reviewing one of the videos. If the participant chose to review a video, I noted which video was selected. Following the completion of the problem, I asked the participant to explain their reasoning for choosing that video. All interviews were audio recorded and transcribed for later analysis.

Videos and Completing the Square

This study used videos about solving quadratic equations via completing the square, a topic that is both challenging for students and often overlooked or underemphasized in the high school mathematics curriculum in the United States (Vinogradova & Weist, 2007). I predicted that this challenging topic would illicit gaps in student learning that would help to make the implicit expectations of participants explicit according to the TDS (Brousseau et al., 2014). I posited that using a topic that was more challenging to participants might allow them to report on their own confusion and result in the disclosure of implicit responsibilities Additionally, when a quadratic is presented in the form that results from completing the square it is easy to extract certain features about the graph and the form can be used to make connections between symbolic and graphic representations (Bossé & Nandakumar, 2005). Making connections between symbolic and graphic or visual representations would allow me to gain additional information about how participants are using videos to learn.

Videos for this study were selected from extant videos about completing the square use it a Google search of the terms “completing the square.” Even though many types of videos could have been used, video lecture is still the most widely available type of video as professors and teachers transfer their in-person lectures into digital formats (Crook & Schofield, 2017; Hegeman, 2015; Ilioudi et al., 2013). From my pilot study, I knew that Khan Academy videos were the default choice for students, so I chose a video by Khan Academy and two others that differed in their mathematics presentation.

The videos that I chose for this study differed slightly in the order of the algebraic steps involved in completing the square. While most mathematically sophisticated watchers would consider these processes to be algebraically equivalent, the study participants did not, and these slight differences became salient to the findings of the study. For this reason, I will call these individual order of steps in each video different processes. By this convention I will say that the three videos selected illustrate three different processes for the method of completing the square. When a participant makes claims and refers to a specific order for the steps in a video I will call it the process of completing the square. Additionally, I will refer to the process of completing the square that participants report that their teacher taught them as the canonical process for completing the square or the canonical process for short.

Data Analysis

Prior to analysis of the interview data, I analyzed data from the MRBQ data and the pre-test about solving quadratic equations. I sorted participants based on their scores on Factor 3 (Beliefs about mathematics as a learnable subject) of the MRBQ. I scored the prior knowledge quiz for correctness and totaled the scores for each participant. However, after analysis of the data, overall prior knowledge level and MRBQ score were not determining factors for my results. I noted that some participants in each of these groups I had constructed had prior knowledge of completing the square, as evidenced by their ability to solve one of the problems on the pretest using this strategy. I sorted the participants again by their prior knowledge of completing the square as shown in Table 4.

Table 4. Participant Matrix for Main Study

Prior Knowledge of Completing the Square	No Prior Knowledge of Completing the Square
Andre	Ben
Anne	Duane
Carrie	Dustin
Connor	Elena
Donna	Ivy
Gail	Jaquelyn
Kim	Octavia
Kristen	Ophelia
Lucy	Simon
Mark	
Marilyn	
Naomi	
Sonia	
Teri	

I coded data for this study beginning with known responsibilities identified by PLACEHOLDER TEXT FOR CONTENT CONTROL and adding codes I identified through thematic analysis. Specifically, I added codes related to responsibilities participants attached to themselves and to the video or instructor in the video. If at any time I found inconsistent evidence between participant claims I rejected the original hypothesis about that participant's responsibility and only kept hypotheses that were supported consistently. I continued to read the data to code for responsibilities and rewrite my hypotheses until all codes were exhausted. In the end I had a complete list of responsibilities held by the participants in my study. After identifying the responsibilities held for participants I reread the interview transcripts looking for patterns in the data, especially related to different levels of prior knowledge of completing the square and especially looking at responsibilities participants held as they were watching each video and how these responsibilities different between the videos.

Results

After completing the coding and reading of my data, I identified a theme that occurred multiple times throughout the data. Participants seemed to be looking for a reproducible process for solving the problem. For those that had prior knowledge of completing the square, this process was the specific steps given to them by their teacher or professor that I will call "the canonical process" (Buchbinder et al., 2019). In the case of this study, the canonical process was shown by the Math Meeting video. When participants were shown a different set of steps or the same steps in a slightly different order, they identified this process as confusing and often defaulted to the process that was originally given by their teacher. This pattern held for participants of all levels of overall prior knowledge and MRBQ level. However, participants without prior knowledge of completing the square did not express that they were looking for a particular process for completing the square and instead attempted to make sense of the set of steps presented in each video.

Participants with some prior knowledge of completing the square also attempted to use the process described in the video. However, when stuck or unsure, these participants returned to the canonical process, often identifying this set of steps as one they were taught or knew previously and the other process as "confusing." Participants seemed to have a didactic contract with their classroom teacher in which the teacher was responsible for organizing mathematical truth in a standard way (Brousseau et al., 2014). Participants, as students in the face-to-face classroom, seemed to be responsible for reproducing this canonical process for completing the square when problem solving. This contract was broken, and subsequently made explicit, when one of the video instructors showed a different process for completing the square, causing the participant to express confusion and to default to the canonical process, the set of specific steps originally given to them by their teacher. If the participant was not taught the canonical process (i.e., had no prior knowledge of completing the square), they did not experience the didactic contract for this particular topic and so did not express the same level of confusion related to seeing a different process for completing the square.

Claim 1: Participants who had some prior knowledge of completing the square express a responsibility to use the process in which they were instructed by their teacher/professor (the canonical process) when shown how to complete the square using a different set of steps.

During the video watching sessions when participants watched the Khan Academy video showing a slightly different process of completing the square, participants often commented that this process was not what their teacher had taught them. Gail, Kim, and Sonia were all participants who had prior knowledge of completing the square and they each expressed that the process for completing the square shown in the Khan Academy video was different than the way in which they were instructed by their teacher. For example, throughout her explanation of the process of completing the square, Gail defaulted to her teacher's process, even though she had just watched a slightly different process in the Khan video. She explained as follows:

> Gail: Yeah. I would've taken the 2X divided it in half and then squared it and then added it to both sides.
>
> Interviewer: Okay, okay, that's fine. And then how did you choose that number?
>
> Gail: I was just taught how to do that.
>
> Interviewer: That's fine. Okay, and then how do you know how to rewrite the equation as X minus something in parentheses down there?
>
> Gail: I guess I just know it from learning it.
>
> Interviewer: Okay, that's fine. Okay, and then why do you have to take the square root of both sides?
>
> Gail: Because there can be two solutions for X, because the original equation is X squared… Because that's just how I was taught it.

Although I was asking questions related to what she had just seen in an instructional video, Gail responded with the set of steps with which she was familiar, the canonical process. She even added that this was the way in which she was taught to complete the square by her teacher. Even though Gail expressed previously that she valued multiple ways to solve a problem, stating that "I think it was important that there are other things out there where you can learn how to do certain math problems, other than what your teacher teaches you," she does not value seeing or using a process that is different from the canonical process for completing the square. Gail seems to differentiate between seeing different ways to find the zeros or roots of a quadratic function, something she values, and seeing different processes for the same strategy as completing the square.

Kim and Sonia, who also had prior knowledge of completing the square, expressed similar ideas about using their teacher's process for completing the square. This responsibility only became explicit when they were shown a different process and they got stuck or were unsure of how to explain the procedure used in the video. When they were shown a different process for completing the square that used different steps or steps in a slightly different order than those taught by their teacher, they referred back, and compared it, to the process that they already knew. I interpreted this as the participant's responsibility to use the teacher's process even when they were shown a video with a different process.

Claim 2: When participants with prior knowledge of completing the square saw a video showing the canonical process, they acknowledged it as the way they knew and/or used it to explain the problem.

In contrast to the previous claim, participants with prior knowledge of completing the square often commented that the process shown in the Math Meeting video was the way that they knew how to solve or that this was the way their teacher had taught them. Kristen, and Mark each expressed their recognition of the strategy shown in the Math Meeting video as that of their teacher. Kristen explained that,

> Kristen: Yeah, we're actually learning this in class right now, so I kind of understand it more… He solved it this way, the way that we're learning in class, how you find out *a*, *b*, and *c*, then you have to divide by *a* and then multiply by *b* over two.
>
> Interviewer: So, this looked more familiar to you than the other video did?
>
> Kristen: Yes, way more… I don't really know cause we're doing this in class right now. I don't know. guess the whole thing, just the way you solved it… That's just what you're supposed to use.

Not only did she identify this strategy as familiar to her, but she also added that it was the one "you're supposed to use," adding a moral imperative to the use of this process.

Mark reported his reasoning for fast forwarding while watching the Math Meeting video by stating that he had identified the process as "what I learned." He went on to explain that,

> I knew this guy was doing it the same way that I was taught to do it. I was like, okay, I'm going to try by myself and get as far as I can. This guy was really far behind. I think I had the right answer. It seemed like when I looked at my work and what he was doing, and we have the same thing. I just wanted to fast forward and get to the end and see if I was correct and that's why I fast forward.

Mark had completed the problem using the canonical process and felt confident with his answer, so he did not need to watch the entire instruction but rather skipped forward to check his answer. All of these participants expressed that the set of steps shown in the Math Meeting video was the process for solving completing the square that they had learned in class, i.e., the canonical process. They then used this set of steps to explain how to solve the problem.

Claim 3: Participants who had some prior knowledge of completing the square found the different procedure to be "confusing."

Mark, Sonia, Donna, Lucy and Anne had prior knowledge of the canonical process for completing the square. When they watched the Khan Academy video that showed a slightly different process, they expressed that the process shown was "confusing." For example, Mark was able to identify the specific difference in the steps that caused him confusion, stating,

> Mark: Yeah. So, I'm used to the way my professor teaches. That's why I was confused about this whole thing… Well, it was a lot different from the way I learn. That's why I was having a hard time.
>
> Interviewer: Okay. It was different from what your professor showed you?
>
> Mark: Yeah.
>
> Interviewer: What parts of it were different?
>
> Mark: The part where he did this part where he didn't add the eight to the other side. What I was taught was to move the eight, or the constant without a variable, to the right side and solve from there. But this guy didn't do it. That's the reason why I had to go back and watch this whole part. I had absolutely no idea what he was talking about.

Mark expressed confusion with the order in which the process of completing the square was shown in the Khan

Academy video. His process moved the constant to the right side of the equals sign prior to any other algebraic manipulations whereas that instructor in the Khan video did not move the constant until much later in the procedure.

Sonia defaulted to using the canonical process to explain completing the square. She summarized what she was experiencing as she watched the Khan video by stating that,

> If I already know something or they're doing it in a method like my teacher didn't teach me, because they do it the way like I wasn't taught, so I didn't want to confuse myself. I was like let me just see if I can get the same answer doing what I already know how to do, right, rather than mess with it. Because sometimes if you watch another way of doing it, it's more confusing than... because when you're doing steps in your head you mix them up or something? So, you just try to see if you can get the same answer that they did and I can, so I know how to do it.

Sonia seemed to be trying to avoid confusion by doing the process with which she was already familiar and not attempting the new process shown in the video. She seems to be convinced that she "know[s] how to do it" because she can use the process and does not have a responsibility to learn a process that is different from the one she was taught by her teacher. Sonia's perspective, as well as the perspectives of Mark, Donna, Lucy and Anne, supports the claim that participants with prior knowledge of one process of completing the square did not hold a responsibility to use a different process other than the one they were taught by their teacher or professor.

Discussion

My research question for this study related to the responsibilities participants held while watching a video. I hypothesized that the prior knowledge and mathematics-related beliefs of participants might have impacted the responsibilities held. However, I found these factors were not as important to participant responsibilities as their prior knowledge of the canonical process of completing the square. Participants who had prior knowledge of completing the square seemed to hold the responsibility to use the process taught by their teacher. In terms of the didactic contract, this responsibility could be called a practical rationality Herbst et al. (2011). Because these practical rationalities were part of the didactic contract, they were implicit and difficult to identify. However, the didactic contract has been revealed within the instructional situations of the mathematics classroom (Herbst et al., 2011). In this study when solving quadratic equations, students were responsible to imitate the solution process given to them by their teacher. This responsibility was violated while watching the Khan Academy video, when the instructor showed a slightly different procedure for completing the square. Although this responsibility was usually implicit, it became explicit when it was violated, and participants expressed their desire to use their teacher's process. This responsibility was tied to participants' prior knowledge of the process for completing the square given by their teacher, the canonical process. Participant overall prior knowledge and MRBQ score did not seem to be related to this responsibility because the responsibility to use the canonical process was expressed by students of different levels of these scores. Further research is needed to investigate whether this student responsibility holds for processes at other levels of mathematics.

Implications from this study apply to both mathematics instruction and video design and use in the mathematics classroom. As more teachers use video as an instructional resource and more students seek out videos on their own to review and remediate mathematics content, it is important to understand the responsibilities student hold about the mathematics they are watching on these videos and that these responsibilities may change the ways in which they select and use mathematics instructional videos. Moreover, teachers must understand the responsibilities their students hold and how this can impact the ways in which they are watching videos. When teachers are selecting math videos for their students they must be aware that students will want to see the set of steps that the teacher has shown them. This is especially true if the teacher is requiring students to use the process that is taught in the classroom. Buchbinder et al. (2019) established that teachers held a responsibility to teach the canonical process for solving equations in one variable. Moreover, researchers found that teachers viewed alternate solutions as unnecessary and potentially confusing. This same type of language was used by participants in the current study when shown videos in which the instructor used a process of completing the square that was not the set of steps they knew. When participants who had prior knowledge of completing the square were shown the Khan or MindYourDecisions video, they expressed that the process was confusing and some even had a desire not to watch the video. I posit that students' responsibility to use the canonical process may be tied to a responsibility held by their teacher. Further research is needed to identify the connection between these responsibilities. However, if we assume that teachers hold this responsibility to the canonical process, then they must also understand that their students also hold this responsibility and that it will impact the ways in which they watch videos both in and out of the classroom.

Teachers may also want to encourage flexibility and the use of multiple representations in their classrooms. The use of representation and solution process flexibly has been identified as a factor that impacts students' ability to solve problems with accuracy and perseverance (Heinze, Star & Verschaffel, 2009). I suggest that a teacher who wants to encourage flexibility and multiple representations must be willing to show multiple sets of steps and ways to talk about the solution process to their students. For example, teachers could make explicit to students where the formula $(\frac{b}{2})^2$ comes from and how it is connected to taking half of the constant in front of the linear term in the quadratic. While teachers may think that they have shown this to students, it is important to continue to make these connections so that students are able to fluidly and flexibly solve problems using different processes and to see the connections between these sets of steps. If students are carrying the responsibility to solve problems using only their teacher's process from their classrooms into the way they choose and watch instructional videos, they may be rejecting alternate solution processes that would allow them to persevere and be successful in solving problems. Teachers may even want to use videos as a teachable moment, showing a different strategy in class and discussing how this set of steps is similar or different from the canonical process and helping students to make connections between representations (Star et al., 2015). While we may hope that teachers and students begin to value flexibility and multiple representations in their classrooms, this aspect of classroom instruction may not change.

Another implication of this study relates to video creators and designers. I suggest that it is important for video designers to have some familiarity with the range of processes and strategies that students may know when designing mathematics instructional videos. This study suggests that a video designer for mathematics instructional videos must be aware of the canonical process for the topic for which they are creating a video.

This does not mean that they must create a video using only the canonical process. My research suggests that they use the canonical process to make connections to their solution process or representation or show how the steps of the procedure can be varied. Specifically, for the videos used in this study, I would suggest three areas of connection that could be made, one that would help students to understand the use of $(\frac{b}{2})^2$, another related to the use of specific letters for variables in the general form of the equation, and one that will help students to gain a deeper understanding of the factoring of the quadratic into a perfect square.

In the Khan Academy video, the instructor talked about finding the constant to add and subtract to the equation by using the general form of the equation $x^2 + 2ax + a^2 + b$ and then identified $-2x$ in the problem as being the same as $2ax$ in the general form. He then showed how to find the a value by setting $2a$ equal to -2 and solving for a and squaring that value to add and subtract to the equation. This was confusing to some participants in this study because they already knew the canonical process for finding this value using the formula $(\frac{b}{2})^2$. If the video designer had been aware of this process, they could have made a connection between their process and the canonical process, showing that, although the processes were ordered and explained differently, they were actually finding the same value through similar mathematical reasoning and providing students with language to describe differences in the steps. Therefore, I suggest that connections between multiple symbolic and algebraic processes should be made when creating videos for students at this level of mathematics.

Limitations and Additional Questions

This study was limited by a small sample size from one university. Further research is required to determine if the findings in the population hold true for students in other universities, at other grade levels and in other mathematics topics. Additionally, the video watching was limited to the three videos selected and the order of the video watching may have impacted the way in which participants were making sense of the mathematics content in the videos. These limitations lead me to think about opportunities for future research.

Further research may explore if participants would hold the same responsibility to use the canonical process if the videos were assigned by the teacher. Moreover, I am wondering what responsibilities would be identified if the teacher was the one introducing multiple solution processes, either by assigning videos that showed these different processes or by showing the processes themselves during their instruction. I suggest that future research should include a study in which the teacher assigned videos for their students to watch that included different solution processes to see if this made a difference in student responsibilities toward the canonical process. My research raises the question concerning whether the responsibility of students toward the use of one specific algorithm for completing the square was connected to their responsibility to their teacher or whether students had a responsibility to solve the problem using only one process and the canonical process happened to be the process with which they were most familiar and comfortable, so they had a responsibility toward that set of steps.

Another suggestion for future research would be to investigate whether students hold the responsibility to use the canonical process because they knew that they would be assessed by the teacher on this process, meaning, the students believe that any deviation will result in reduced points. I suggest a study that includes an interview with the teachers of student participants as well as an analysis of their assessments to see if the teachers were requiring their students to use a particular set of steps for completing the square as is proposed by Buchbinder et al. (2019). Perhaps an intervention could be introduced to encourage teachers to allow students to explore different processes for solving quadratic functions using completing the square. Student could then be asked to watch videos that show different processes to see if they maintain the responsibility to use the canonical process.

Conclusion

Video watching for mathematics teaching and learning is becoming ubiquitous. Whether for use in the flipped classroom (Bergmann & Sams, 2007; Bowers et al., 2012; Kirvan et al., 2015; Pinzon et al., 2016), fully online instruction (Hegeman, 2015; Zhang et al., 2006) or even as a resource within the traditional face-to-face classroom, students are accessing videos to learn and remediate mathematics content at all levels of instruction (Pinzon et al., 2016). While researchers have investigated student learning gains from and attitudes toward mathematics instructional videos, few studies have observed what students are doing as they watch videos to learn mathematics. This study has begun to identify student responsibilities while watching mathematics instructional videos. Specifically, I have applied the theory of didactic situations to assist in the framing of responsibilities held by college algebra students as they watch mathematics videos that show solution processes for solving quadratic functions by completing the square. Students seemed to hold a responsibility to use the canonical processes for this solution strategy. These findings hold implications for both mathematics educators and video creators for mathematics learning. I hope that future research will be able to identify other responsibilities for students across grade levels and content areas.

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Author Information

Sue Kelley

https://orcid.org/0000-0001-6736-7868

Temple University

1301 Cecil B. Moore Ave

Philadelphia, PA 19122

United States

Contact e-mail: *sue.kelley@temple.edu*

Appendix A. Full Text of Pretest

1. a. What are the solutions of the quadratic equation $4x^2 - 8x - 12 = 0$?

b. What does it mean to be a solution to a quadratic equation?

2. a. What are the solutions to $3x^2 + 12x + 6 = 0$?

b. Are you able to solve this problem by completing the square?

If so, show it here.

$$y = x^2 - 6x + 8$$

3. a. The equation above represents a parabola in the *xy*-plane. Which of the following equivalent forms of the equation displays the x-intercepts of the parabola as constants or coefficients?

A) $y - 8 = x^2 - 6x$

B) $y + 1 = (x - 3)^2$

C) $y = x(x - 6) + 8$

D) $y = (x - 2)(x - 4)$

b. What are the x-intercepts for this parabola?

c. Explain how you decided that these values represent the x-intercepts for this parabola.

$$x^2 + 6x + 4$$

4. a. Which of the following is equivalent to the expression above?

A) $(x + 3)^2 + 5$

B) $(x + 3)^2 - 5$

C) $(x - 3)^2 + 5$

D) $(x - 3)^2 - 5$

b. Explain how you decided that these expressions are equivalent.

5. a.

$$h(t) = -16t^2 + 110t + 72$$

The function above models the height h, in feet, of an object above ground t seconds after being launched straight up in the air. What does the number 72 represent in the function?

b. What do the x-intercepts of the function above represent?

Mathematics-Related Beliefs Questionnaire

Please respond to each statement	Strongly agree	Somewhat agree	Neither agree nor disagree	Somewhat disagree	Strongly disagree
1. Making mistakes is part of learning math.	O	O	O	O	O
2. Group work helps me learn mathematics.	O	O	O	O	O
3. Mathematics learning is mainly memorizing.	O	O	O	O	O
4. The importance of competence in mathematics has been emphasized at my home.	O	O	O	O	O
5. Anyone can learn mathematics.	O	O	O	O	O
6. There are several ways to find the correct solution of a mathematical problem.	O	O	O	O	O
7. I am hard working by nature.	O	O	O	O	O
8. Solving a mathematics problem is demanding and requires thinking, even from smart students.	O	O	O	O	O
9. Mathematics is continually evolving. New things are still discovered.	O	O	O	O	O
10. There is only one way to find the correct solution of a mathematics problem.	O	O	O	O	O
11. Mathematics is used by a lot of people in their daily life.	O	O	O	O	O
12. My family has encouraged me to study mathematics.	O	O	O	O	O
13. I'm only satisfied when I get a good grade in mathematics.	O	O	O	O	O
14. The example of my parent(s) has had a positive influence on my motivation.	O	O	O	O	O
15. I believe that I will receive and excellent grade for mathematics this year.	O	O	O	O	O
16. By doing the best I can in mathematics I want to show the teacher that I'm better than most other students.	O	O	O	O	O
17. I like doing mathematics.	O	O	O	O	O
18. I want to do well in mathematics to show the teacher and my fellow students how good I am at it.	O	O	O	O	O
19. I can understand course material in mathematics.	O	O	O	O	O
20. To me mathematics is an important subject.	O	O	O	O	O

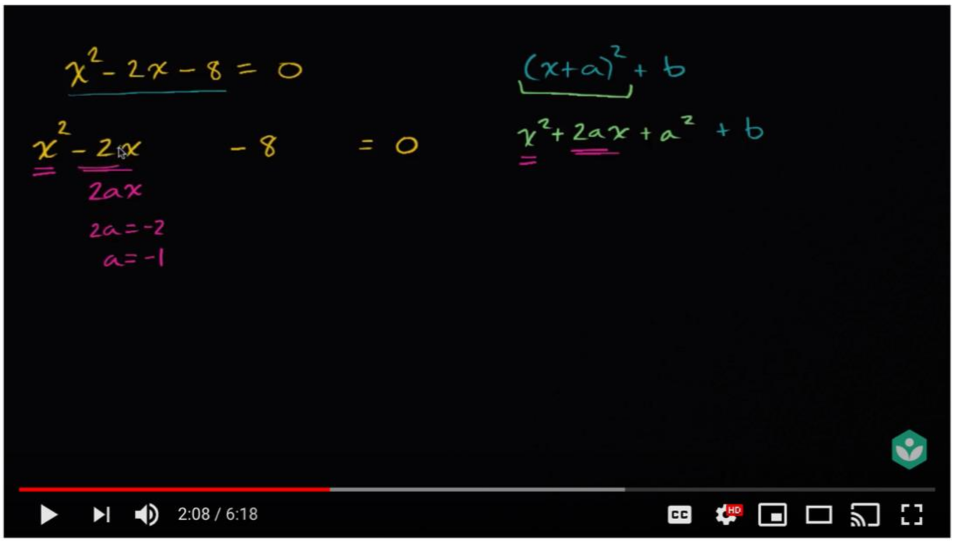
21. I prefer mathematics tasks for which I have to exert myself in order to find the solution.	O	O	O	O	O
22. I know I can do well in mathematics.	O	O	O	O	O
23. If I try hard enough, then I can understand the course material of the mathematics class.	O	O	O	O	O
24. When I have the opportunity, I choose mathematical assignments that I can learn from even if I'm not at all sure of getting a good grade.	O	O	O	O	O
25. I'm very interested in mathematics.	O	O	O	O	O
26. Taking into account the level of difficulty of our mathematics course, the teacher, and my knowledge and skills, I'm confident that I will get a good grade for mathematics.	O	O	O	O	O
27. I think I will be able to use what I learn in mathematics in other courses.	O	O	O	O	O
28. I am no good at math.	O	O	O	O	O
29. Those who are good in mathematics can solve problems in a few minutes.	O	O	O	O	O
30. I am not the type to do well in math.	O	O	O	O	O
31. My parents enjoy helping me with mathematics problems.	O	O	O	O	O
32. It is a waste of time when the teacher makes us think on our own about how to solve a new mathematical problem.	O	O	O	O	O
33. My parents expect that I will get a good grade in mathematics.	O	O	O	O	O
34. I expect to get a good grade on assignments and tests of mathematics.	O	O	O	O	O
35. Math has been my worst subject.	O	O	O	O	O
36. Mathematics enables people to better understand that world they live in.	O	O	O	O	O
37. I have not worked very hard in math.	O	O	O	O	O
38. Mathematics is a mechanical and boring subject.	O	O	O	O	O
39. I can get good grades in math.	O	O	O	O	O
40. I always prepare carefully for exams.	O	O	O	O	O
41. Mathematics has been my favorite subject.	O	O	O	O	O
42. I am sure that I can learn math.	O	O	O	O	O
43. My major concern when learning mathematics is to get a good grade.	O	O	O	O	O

Factor 3 of MRBQ

Prompt Number	Prompt
Factor 3: Beliefs about mathematics as a learnable subject	
16.	Anyone can learn mathematics.
27.	If I work hard, then I will understand the course material of the mathematics class.
29.	Our mathematics teacher appreciates it when we have tried hard, even if our results are not so good.
30.	Our mathematics teacher thinks that errors are okay and can be helpful for learning.
34.	Making mistakes is an important part of learning mathematics.
35.	I can understand even the most difficult material presented in the mathematics course.
36.	Our mathematics teacher tries to make the lessons interesting.
39.	Mathematics is used by a lot of people in their daily life.

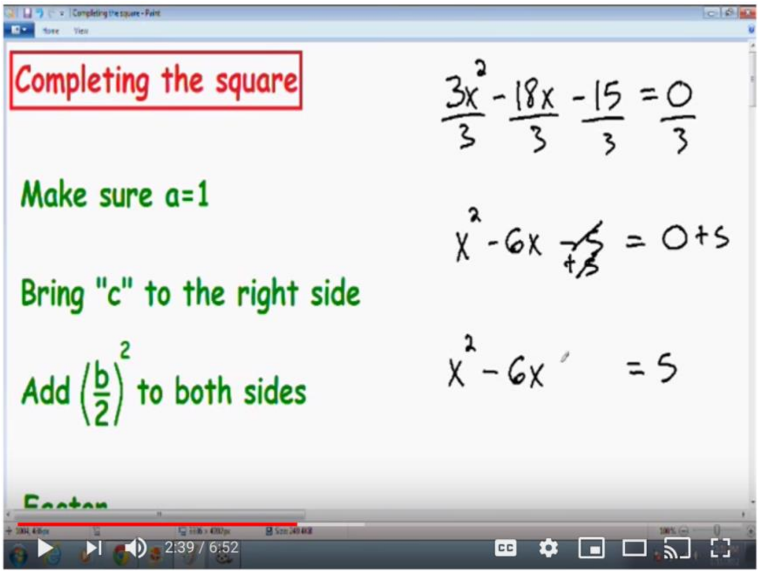
Appendix B. Video Selection

Figure 1. Screenshot of Khan Academy Video



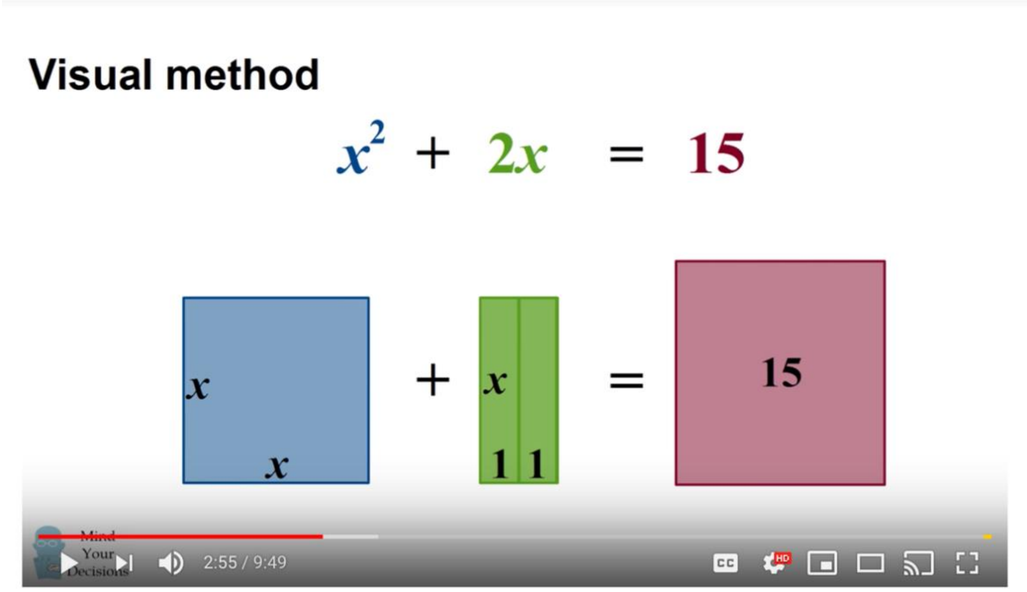
The Khan Academy video, shown in Figure 1, presents a process using a symbolic representation for completing the square. While multiple completing the square videos by Khan Academy are available on YouTube, I selected this video because it showed a different process for completing the square than the Math Meeting video. Beginning with the example equation $x^2 - 2x - 8 = 0$, the instructor shows the general formula, $(x + a)^2 + b$, and explains that this is the goal for completing the square. He then expands this form of the equation to show $x^2 + 2ax + a^2 + b$, explaining that he is working backward to show what the expanded form of the final goal looks like. The instructor next solves for a in the general form by setting the coefficient in the example equation equal to a in the general form ($2a = -2$), squaring the value of a and adding and subtracting this value to the equation. Throughout his process, the instructor explains that he is trying to match the pattern in the general form of the equation. He concludes his solution by explaining that, while there are other processes to solve quadratic equations, completing the square is “very powerful” because it always works. He expresses that the quadratic function comes directly from completing the square but does not attempt to show how this is true.

Figure 2. Screenshot of Math Meeting Video



The Math Meeting video, shown in Figure 2, also has a symbolic representation of the process for completing the square. However, the order for the steps to this process are slightly different and the process includes the use of the formula, $\left(\frac{b}{2}\right)^2$. Additionally, in comparison to the Khan Academy video, this video contains even less explanation about the reasoning behind the process and instead presents a series of steps to be followed. In this video, the instructor begins by explaining that completing the square is not the easiest way to solve this type of problem. He offers factoring or the quadratic equation as other options but then explains that sometimes in math class a student will be asked to solve using completing the square. He then begins with the example $3x^2 - 18x - 15 = 0$, which is different from the example used in Khan Academy video in that it has a leading coefficient that is not equal to 1 and could not be easily solved by factoring. Instead of trying to tie an explanation to the general formula for a perfect square trinomial, as was done in the Khan Academy video, the instructor simply explains a step-by-step procedure for completing the square. The steps for the solution process are shown as a list on the left side of the screen. The instructor annotates the solution on the right side of the screen as he completes each step. He begins by dividing each part of the equation by 3 to make the a value equal to 1. Step 2 involves moving the constant to the right side of the equation by adding it to both sides. In contrast, the instructor in the Khan Academy video does not move the constant to the other side until after completing the square. This difference was noted by participants as something that was different or confusing to them and will be discussed with more detail in the *Results* section of this paper. The next step in the procedure is to add $\left(\frac{b}{2}\right)^2$ to both sides of the equation. The instructor explains that he is doing this because it will make it easy to factor later on. The instructor simplifies and then factors the left side of the equation. He explains that the factors should be the same or you did something wrong and he rewrites the factors as a squared binomial. The next step in this solution process is to take the square root of both sides. The instructor notes that when you have the square root of a constant you must have both a positive and negative answer. He then adds 3 to both sides of the equation to get $\pm\sqrt{14} + 3$. He rewrites the equation as $x = 3 \pm \sqrt{14}$ and declares that this is the final answer. This video does not show the answer as two separate values as is shown in the Khan Academy video.

Figure 3. Screenshot of MindYourDecisions Video



In a similar way to the other videos, the MindYourDecisions video, shown in Figure 3, also talks about the fact that there are other solution processes for solving quadratics and that completing the square is connected to the quadratic formula. However, the main focus of this video is to provide a geometric proof for the quadratic formula in the form of completing the square. The instructor begins with an example equation ($x^2 + 2x - 15 = 0$) and shows how this equation can be reorganized and modeled using rectangles. Each term of the equation is color coded and the colors match to the corresponding rectangle in the model. The instructor explains that the goal of completing the square is to find the value of the missing piece of the rectangle and to add it to both sides. Because each side of the equation contains a square with equal area, the value of x on the left side of the equation can be found by setting the side lengths of each square equal to each other. He also explains that since we are dealing with square roots, we can actually have a value for the side length that is a negative number. The instructor comments that this seems strange in terms of the geometric model but works mathematically. After completing the model using the specific example, the instructor repeats the proof using a general form of the equation ($ax^2 + bx + c = 0$). I have chosen to stop the video at this point in order to make the content more similar to the Khan Academy and Math Meeting videos. During the interview, I stopped the video at the point where the instructor began to talk about the quadratic formula. Even so, this video provides a contrast to the other videos as it showed a visual, geometric representation of completing the square that student may find valuable in comparison to the symbolic representation of the Khan Academy and Math Meeting videos.